The Ticker-Tape Interpretation of Quantum Mechanics

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The Ticker Tape Interpretation of Quantum Mechanics is a Kantian based interpretation, similar to Copenhagen. It examines the everyday properties of measurements and shows that they lead inexorably to Quantum Mechanics as we know it. The classical Quantum Mechanic formalism is derived. Some conjectures are made about the nature of time which allows the above results to be applied generally.

Author's Note

The author has rather playfully borrowed the title of some of Einstein's famous "Principles". Even though the principles in this paper have the same names as those found in Special or General Relativity, *they have nothing to do with any specifics of Special or General Relativity*; the names were chosen however because, at some level, the both versions derive from an even more general principle.

1. Why interpret?

An interpretation of Quantum Mechanics is an explanation of why Quantum Mechanics is the way it is.

There are some who do not believe in the need for interpretation. Richard Feynman famously Feynman famously stated that "no-one understands Quantum Mechanics" and criticised those who felt that the Universe needed to be simple or beautiful. To Feynman, the Universe is the way it is. If a theory is in 100% harmony with observation, then the theory should be accepted as it is. If it is logically consistent, fits the facts but does not appear to make sense, that's too bad.

However there are some good reasons to search for interpretations.

1. Humans need to find an explanation that they personally feel comfortable with. Manipulating equations without understanding is just not satisfying enough. There is a psychological need for interpretation. Psychology however has proven to be a poor guide in the interpretation of Quantum Mechanics

2. Philosophical principles historically have been at the heart of the development of Physics – for example, Galileo's Principle of Relativity was used by Galileo and Newton to overcome Aristotelian Physics and replace it with Classical Newtonian Mechanics. Later it was used by Einstein in the Special Theory of Relativity. Mach's Principle was instrumental in Einstein's thinking leading towards General Relativity.

3. Physics is unfinished business. Research into topics such as Quantum Gravity has reached the point where it is not possible to directly test theories and has led theoretical physicists to again become directly involved with philosophical considerations, up to and including "What constitutes Science?"

   Interpretations should increase our understanding of Physics and point the way forward towards new and better theories. Despite Feynman's comments, philosophy and a sense of beauty has had a profound effect on the development of Physics, and the writings of all great scientists, including Feynman's, show a deep appreciation of such things

2. The Halting Problem

Quantum Mechanics is supposed to be, or lead to, the ultimate "Theory of Everything". What would a "Theory of Everything" look like?

If theory A is the "Theory of Everything" but uses concept B, should we then pursue an explanation for concept B? When do we stop? What if B is something we are not comfortable with?

Should we accept a "Theory of Everything" which explains everything in terms of quarks and leptons and 4-dimensional space-time, or should we seek to explain the behaviour of quarks, leptons and space-time? If we could explain quarks and leptons in terms of strings, should we seek to explain the properties of strings? When do we stop? What should we accept as fundamental?

Alternatively we can turn to philosophy. Immanuel Kant, for example, described a world where there is a distinction between the mind and the external world, and between information drawn from the senses and information resulting from the application of logic. Should we expect a "Theory of Everything" to reflect philosophy? Arguably the Copenhagen Interpretation does; it is based on Positivism.

3. Philosophical Underpinnings

This paper adopts a world view that is essentially Kantian. The Kantian world view consists of an external world that is perceived through our sensors; we use the information provided to build mental models of the external world but cannot know its true nature.

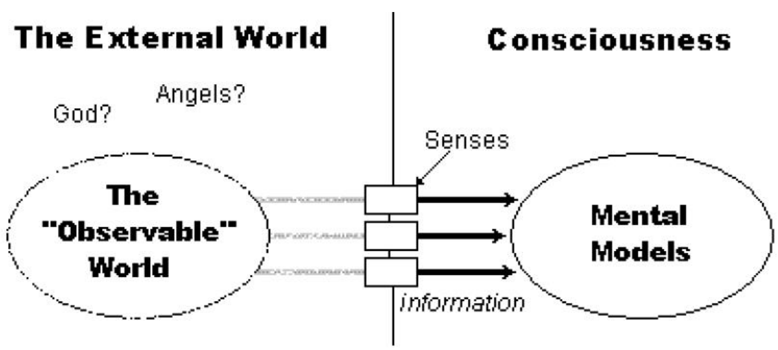

Figure 1.
Kantian World View (circa 1785).

Copenhagen clarifies aspects of the Kantian world view. (It has been argued that the Copenhagen Interpretation is Positivist, however subtle arguments over philosophical classifications are beyond the scope of this paper) Quantum Mechanics replaces the vague idea of sensory input with that of measurement and precise mathematical description. The idea of a conscious agent is replaced by intelligence, which need not be human (although some commentators would dispute this). It could, for example, be a robot.

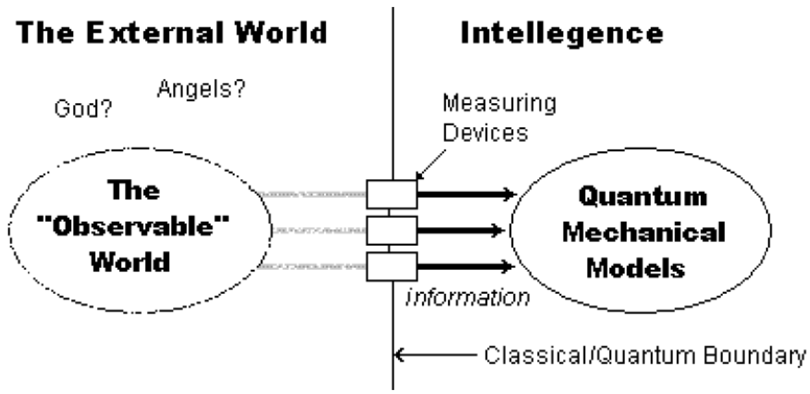

Figure 2.
Copenhagen World View (circa 1928).

The Copenhagen Interpretation acknowledges that our mental models may be incomplete; it may be necessary to apply a wave model in some situations, and apply a particle model in another (Complementary Principle). Realism, on the other hand, is rooted in the belief that the world is as it appears, and seeks to preserve macroscopic models, possibly beyond their domain of applicability.

4. Measurement

Measurements are regarded as "elements of reality", although no definition of what this means is given. The observer is logically separate from the measurement itself. A measurement discontinuously changes the probability distribution associated with a "physically significant" random variable.

Definition: A measuring device that produces a single real number as its output is called a "basic" measuring device, and the measurements it produces are called "basic" measurements. The notation $\mathbf{X} = x$ is used to mean the device **X** has been used to make a measurement and result has been reported as x.

Why a real number? Measurement results can be put "in order" from the smallest to the largest. I.e. If $\mathbf{M}$ = {set of possible measurement values}, then there is a strict total ordering of **M**, which creates an order preserving mapping from **M** to some subset of Real numbers.

It is postulated that Quantum measurements are basic measurements, or logical combinations of such measurements. For example, ($\mathbf{X}=x_1$ or $\mathbf{X}=x_2$) is a possible quantum measurement, as is ($\mathbf{X} > x_1$) and ($\mathbf{X} \neq x_2$)

Definition: A history is a sequence of measurements and denoted $\mathcal{H} = (\mathbf{m}_0, \mathbf{m}_1, \mathbf{m}_2, ..., \mathbf{m}_n)$ where $\mathbf{m}_i$ are measurements. If all measurements are "basic", the history will look like ($\mathbf{X} = x_0$, $\mathbf{Y}_1 = y_1$, $\mathbf{Z}_2 = z_2$, ..., $\mathbf{W}_n = w_n$).

5. The Equivalence Principle

There is no transition from the Quantum world to the Classical world. The difference between measurement in the Classical world and measurement in the Quantum world is a matter of interpretation.

Quantum Mechanics is a mental model; the waveform and any measurement operator **A** are mental constructions, built on top of the information gathered from "raw" measurements. Classical mechanics is also a mental model (since we know it is not "true", it cannot be otherwise); typically Classical Mechanics deals with the expected value $<\mathbf{A}>$ and regards the difference from $<\mathbf{A}>$ to be “error”.

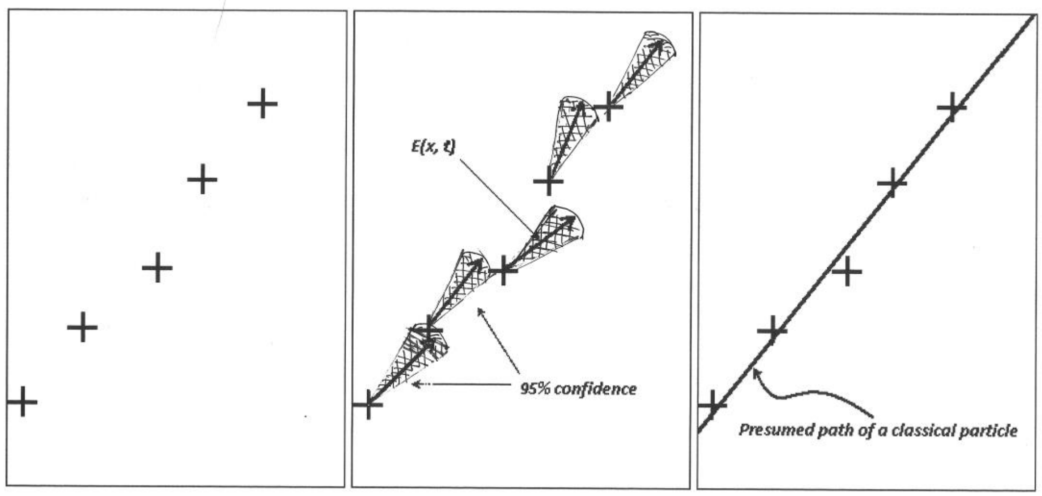

Figure 3
Measurements and Mental Models.

In the diagram above, the left pane shows the “raw” position measurements of a particle moving diagonally from left to right is shown. The central pane *adds* the intellectual machinery of Quantum Mechanics; the *calculated* 95% confidence intervals are shown in gray. The right pane *adds* the intellectual machinery of Classical Mechanics; the presumed classical path is shown in red.

In Quantum Mechanics, the uncertainty in the position of a particle created by Heisenberg Uncertainty Principle is viewed as intrinsic to the system; the measurement themselves are taken at face value (*The Principle of Exact Measurement*).

In Classical Mechanics, it is the measurement that contains the “error” (uncertainty) in the position of the particle; the “error” has a multitude of sources external to the system itself, typically related to the construction of the measuring devices and lack of knowledge of initial conditions, but there is always a presumption that if these influences could be eradicated, exact measurements would be possible and that the predictions of Classical mechanics would be confirmed.

6. Mach Devices

Bohr expressed the opinion that measuring devices are “essentially classical” in that they measure classical quantities such as time and space [1]. This implies knowledge of classical mechanics is required before Quantum Mechanical measurements can be understood, yet Quantum Mechanics is presumed to be more fundamental than classical physics. It is also not easy to apply Bohr's vision of essentially classical measuring devices to abstract concepts such as QCD colour.

Rather than follow Bohr, we take a slightly different view of measurement.

Definition: A measuring device is a Mach device with respect to an observer if the following applies:

(i) The device produces a stream of basic measurements recorded on a ticker-tape (or equivalent) accessible to the observer. Measurements are recorded in the order they are made.

(ii) The measurements are "repeatable".

(iii) The device's internal structure is unknown. It is a "black box". There is no *a-prior information* about what the numbers it produces mean.

(iv) The observer does not have access to a clock.

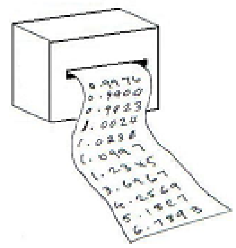

Figure 4.
A Mach device.

A Mach device is the most primitive measurement device possible: it only produces basic measurements.

Note that a classical measuring device equipped with a digital output *generally* does not qualify as a Mach device. As remarked previously, Classical Mechanics presumes that the measurement **X**=*x* really corresponds to a probability density function for the "true" value of **X** with expected value E(**X**) = <**X**> = *x*, even if that probability function is unknown in detail.

A typical set of probability distributions associated with a classical measurement is shown in figure 5.

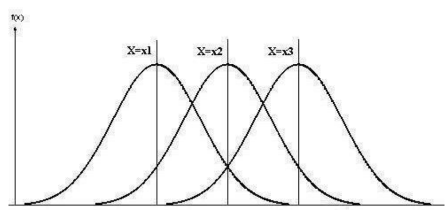

Figure 5.
Probability distributions for numeric "labels".

If there is overlap between the probability density functions (as is the case for the probability density functions for $\mathbf{X} = x_1$ and $\mathbf{X} = x_2$ in figure (5)), repeated measurements of the system may result in the value x_1 or x_2. I.e. if there is any overlap in the probability density functions, the result of any measurement of **X** is not repeatable and so the measuring device is not a Mach device.

7. Principle of Relativity (Mach's Principle)

The output from a single Mach device is meaningless.

This is a generalisation of Mach's Principle. Mach's statement of the Principle of Relativity famously influenced Einstein but the principle itself dates back at least to Galileo. Ernst Mach argued that it would be meaningless to talk about the motion of a single particle in an empty Universe. All motion is relative. In fact, all measurement is relative. If there is no context, a measurement stream becomes a meaningless stream of numbers.

8. Mach Banishes Determinism

Suppose $\boldsymbol{X} = f(t)$ is a classical quantity, and **X** is a measuring devices that faithfully returns $\boldsymbol{X}$. If **X** is a Mach device with respect to an observer, it is not possible for that observer to determine what the device measures from the measurement history. Why? There is no way to calibrate the device.

Suppose we construct a second device **Y** whose output is related to the first by

$$\mathbf{Y} = \zeta\,(\mathbf{X}(t))$$

The second device is sealed, mixed up with the first and given to the naive observer so they both become Mach devices. Which measures the "fundamental" quantity? **X** and **Y**? **X**? **Y**? In fact, we can build the device **Y** so it returns any measurement profile we like.

If a measuring device qualifies as a Mach device ***except*** *that it is known that it measures a quantity* **X** *where* $\mathbf{X} = f(t)$ *for some function f , then the output from that device is meaningless.*

9. State

<u>*Definition:*</u> A system state is *any* representation Ψ such that there is a rule for the calculation of the probability $P(\mathbf{x}\,|\Psi)$ where **x** is a measurement outcome and $P(\mathbf{x}\,|\Psi) = P(\mathbf{x}\,|\,\mathcal{H})$ for all**,** where $\mathcal{H}$ is the known history of the system.

The minimal sub-history H of $\mathcal{H}$ such that $P(\mathbf{x}\,|\,H) = P(\mathbf{x}\,|\,\mathcal{H})$ for all **x** is one possible system state representation**.**

The test of a good theory is whether it can make accurate predictions. In the case of Quantum Mechanics, the obvious question is: How much history is necessary before an observer can make accurate predictions?

The answer cannot be that the observer must know the entire history of a system since the beginning of time since that information will never be available. So what are the alternatives? One possibility is to only count the last N measurements for some N, perhaps giving the more recent measurements more "weight". But how do we chose N? Why is one choice of N better than another? How do we assign "weights" to newer and older measurements?

Given the problems of choosing any special value for N, it makes sense that the probability of any measurement **X**=x depends only on the last value of each measuring device in the system.

10. Transition Probabilities and Repeatability

Measurements are expected to be repeatable. I.e. if two measurements are made, one immediately after the other, the results of the two measurements should agree.

Mach devices do not come equipped with a clock. There is no sense of time. It is not possible to know what the interval between any two measurements is. If the results of two consecutive measurements disagree it could be that measuring device is a working Mach device and the interval was so long that the system slowly evolved into another state. It is also possible that the device is "faulty" (not a working Mach device). But it is not possible to tell which is the case.

Since we have no clock we restrict our attention to systems with stable state transition probabilities. I.e. $P(\Psi|\Psi) = 1$ for any state Ψ. I.e. From the ticker tape, select out measurements of the form ($\mathbf{B} = b$, $\mathbf{A} = ?$) and calculate the proportion that have $\mathbf{A} = a_i$.

11. How do we know what we are measuring?

If the stream of measurement is deterministic, then the output from the device is meaningless. It seems that a measurement stream can only be "understood" if it contains a random component.

The demand for Repeatability requires that the randomness cannot occur between uninterrupted repeated measurements from the same measuring device. That only leaves one place where true randomness can exist.

Fortunately early 20th Century scientists have done most of the work for us.

Definition: **A** and **B** are conjugate devices if a measurement of **A** "wipes out" the predictive power of the measurement of **B**, and a measurement of **B** "wipes out" the measurement of **A**. More precisely

$$P(\mathbf{x} \mid (\mathbf{A}=a, \mathbf{B}=b)) = P(\mathbf{x} \mid \mathbf{B}=b) \text{ for all } \mathbf{x}, a, b$$

and

$$P(\mathbf{x} \mid (\mathbf{B}=b, \mathbf{A}=a)) = P(\mathbf{x} \mid \mathbf{A}=a) \text{ for all } \mathbf{x}, b, a$$

where the probabilities $P(\mathbf{X}|\mathbf{Y})$ are derived via an analysis of a large number of previous measurements.

Corollary: The last measurement provided by a set of conjugate devices is a representation of the state of the system described by the devices.

We assert that it is possible to assign meaning to a measurement stream provided that

(i) The measuring devices associated with the measurement stream are conjugate.

(ii) The state transition probabilities are symmetric with respect to the start and end states.

A physical system, measuring devices and observers that satisfy the condition above is collectively called a quantum system.

12. The Formalism of Quantum Mechanics

12.1 Feyman's Rules

We follow the argument put forward by Ariel Caticha[2].

Definition: If $\mathcal{H}_A = (\mathbf{m}_1, \ldots, \mathbf{m}_k)$ and $\mathcal{H}_B = (\mathbf{m}_{k+1}, \ldots, \mathbf{m}_{k+m})$ are histories, then

$$\mathcal{H}_A \wedge \mathcal{H}_B = (\mathbf{m}_1, \ldots, \mathbf{m}_k, \mathbf{m}_{k+1}, \ldots, \mathbf{m}_{k+m})$$

The history $\mathcal{H}_A$ must "follow" the history $\mathcal{H}_B$ and not overlap in time. The operator $\wedge$ is read as "and".

Definition: Let $\mathcal{H}_A$ be a history, then $\mathcal{H}_B$ is an alternative history to $\mathcal{H}_A$ if $\mathcal{H}_B$ is the same as $\mathcal{H}_A$ except that some of the measurements, other than the initial and final measurements, have different values.

Definition: If $\mathcal{H}_A = (\mathbf{m}_1, \ldots, \mathbf{m}_{Ai}, \ldots, \mathbf{m}_n)$ and $\mathcal{H}_B = (\mathbf{m}_1, \ldots, \mathbf{m}_{Bi}, \ldots, \mathbf{m}_n)$ are alternative histories which differ in the value of the i^{th} measurement, then

$$\mathcal{H}_A \vee \mathcal{H}_B = (\mathbf{m}_1, \ldots, \mathbf{m}_{Ai} \vee \mathbf{m}_{Bi}, \ldots, \mathbf{m}_n)$$

The operator $\vee$ is read as "or".

If $\mathbf{m}_{Ai}$ is the measurement $\mathbf{X} = x_i$, and $\mathbf{m}_{Bi}$ is the measurement $\mathbf{X} = x_2$, then

$$\mathcal{H}_A \vee \mathcal{H}_B = (\mathbf{m}_1, \ldots, \mathbf{X} \in \{x_1, x_2\}, \ldots, \mathbf{m}_n)$$

and $\mathbf{X} \in \{x_1, x_2\}$ is regarded as a measurement in its own regard, derived from the basic measurements $\mathbf{X} = x_1$ and $\mathbf{X} = x_2$.

The operators $\wedge$ and $\vee$ obey the following relations:

$$a \vee b = b \vee a$$
$$(a \vee b) \vee c = a \vee (b \vee c)$$
$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$
$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

If $\mathcal{H}$ = set of possible histories for a system, the any representation $(\Psi, +, \times)$ with $\Psi: \mathcal{H} \to \Omega$ where Ω is some algebraic system, over a field F, and

$$\Psi(a \vee b) = \Psi(a) + \Psi(b)$$
$$\Psi(a \wedge b) = \Psi(a) \times \Psi(b)$$

would carry these properties across.

Caticha[2] shows that standard addition and multiplication fit the requirements for + and $\times$.

12.2 Derivation of Standard Equations of Quantum Mechanics

The mapping above assumes that each possible outcome is equally likely. We want to describe the case where one state $\mathbf{\Psi'}$ is more (or less) likely than the other state - it seems sensible to try to represent $\mathbf{\Psi'} = (\boldsymbol{\lambda\Psi})$.

1) Feynman's Rules imply that state has an algebraic structure generated by the union and concatenation of histories (and therefore states). In particular,

$$P(\mathbf{\Psi} + \mathbf{\Psi} \,|\mathbf{\Phi}) = P(\mathbf{\Psi} \vee \mathbf{\Psi} \,|\mathbf{\Phi}) = P(\mathbf{\Psi} \,|\mathbf{\Phi})$$

and

$$P(\mathbf{\Phi}|\, \mathbf{\Psi} + \mathbf{\Psi}) = P(\mathbf{\Phi}|\, \mathbf{\Psi} \vee \mathbf{\Psi}) = P(\mathbf{\Phi}|\, \mathbf{\Psi}).$$

This implies

$$P(\boldsymbol{\lambda\Psi}) = P(\mathbf{\Psi}) \text{ where } \boldsymbol{\lambda\Psi} = \mathbf{\Psi} + \ldots + \mathbf{\Psi} \ (\boldsymbol{\lambda} \text{ times) for all } \boldsymbol{\lambda} = 1,2,3,4\ldots$$

and

$$P(0.\mathbf{\Psi}) = P(\phi) = 0$$

This result can be extended to

$$P(\boldsymbol{\lambda\Psi}) = P(\mathbf{\Psi}) \text{ where } \boldsymbol{\lambda} \in F. \qquad (10.2.1)$$

2) Probability is not linear in state. If this was so then suppose $P(\mathbf{\Phi} \mid \mathbf{\Psi}) > 0$. It follows

$$P(\mathbf{\Psi} + \mathbf{\Phi} \mid \mathbf{\Psi}) = P(\mathbf{\Psi} \mid \mathbf{\Psi}) + P(\mathbf{\Phi} \mid \mathbf{\Psi}) = 1 + P(\mathbf{\Phi} \mid \mathbf{\Psi}) > 1$$

which cannot be correct.

3) The numbers $\mathbf{\Psi}$ and $\mathbf{\Phi}$ are not members of the field F**.** If this was not the case, then $\mathbf{\Psi} = \alpha\mathbf{\Phi}$ for some number α, and so

$$P(\mathbf{\Psi} \mid \varphi) = P(\alpha\mathbf{\Phi} \mid \varphi) = P(\mathbf{\Phi} \mid \varphi)$$

which cannot be correct.

4) If $\mathbf{\Psi}_i$ are the states associated with distinct outcomes of a measurement, then

$$P(\mathbf{\Psi}_i \mid \mathbf{\Psi}_k) = \delta_{ik} \qquad (10.2.2)$$

The above expression looks a lot like an inner product, except that probability is always positive.

Putting it all together, (10.2.1) strongly suggests that the probability is a function of $\frac{\mathbf{\Psi}}{|\mathbf{\Psi}|}$ rather than $\mathbf{\Psi}$. I.e.

$$P(\mathbf{\Psi}) = g\left(\frac{\mathbf{\Psi}}{|\mathbf{\Psi}|}\right)$$

(10.2.2) strongly suggests that the presence of an inner product

$$P(\mathbf{\Psi}_i \mid \mathbf{\Psi}_k) = f\left(\left\langle \frac{\mathbf{\Psi}_i}{|\mathbf{\Psi}_i|}, \frac{\mathbf{\Psi}_k}{|\mathbf{\Psi}_k|} \right\rangle\right) \qquad (10.2.3)$$

where $<.|.>$ is an inner product and f: $\mathbf{R} \rightarrow [0,1]$ with $f(-1) = 1$, $f(0) = 0$ and $f(1) = 1$.

Choosing $f(x) = |x|^2$, probably the simplest possible choice, gives the “correct” QM formalization.

The standard representation of measuring devices as operators and measurement values as eigenvalues follows.

12.3 Some Examples - Algebraic Extensions

It is sometimes asked why complex numbers are so prevalent in Quantum Mechanics. They occur as the solution to algebraic equations that arise in the calculation of representations

Assume **A** and **B** are conjugate devices that both return two values ↑ and ↓.

Let $\Psi_{\mathbf{A}\uparrow} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\Psi_{\mathbf{A}\downarrow} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, then finding a representation for $\Psi_{\mathbf{B}\uparrow}$ and $\Psi_{\mathbf{B}\uparrow}$ is equivalent to solving the equations

$$\Psi_{\mathbf{B}\uparrow} = a\Psi_{\mathbf{A}\uparrow} + b\Psi_{\mathbf{A}\downarrow} \qquad \text{for some } a,b$$
$$\Psi_{\mathbf{B}\downarrow} = c\Psi_{\mathbf{A}\uparrow} + d\Psi_{\mathbf{A}\downarrow} \qquad \text{for some } c,d$$

subject to $P(\Psi_{\mathbf{A}i} \mid \Psi_{\mathbf{B}j}) = 0.5$ for $i\ j$ in $\{\uparrow,\downarrow\}$. Solving yields (non-uniquely)

$$\Psi_{\mathbf{A}\uparrow} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \Psi_{\mathbf{A}\downarrow} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \Psi_{\mathbf{B}\uparrow} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \Psi_{\mathbf{B}\downarrow} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

which leads to an obvious geometric interpretation.

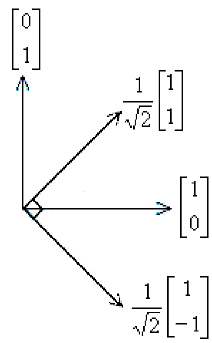

Figure 6
Geometric Interpretation of state space

If a 3rd 2-valued conjugate device **C** is introduced, then finding a representation is again a case of finding

$$\Psi_{\mathbf{C}\uparrow} = a'\Psi_{\mathbf{A}\uparrow} + b\Psi_{\mathbf{A}\downarrow} \qquad \text{for some } a,b$$
$$\Psi_{\mathbf{C}\downarrow} = c\Psi_{\mathbf{A}\uparrow} + d\Psi_{\mathbf{A}\downarrow} \qquad \text{for some } c,d$$

subject to $P(\Psi_{\mathbf{C}i} \mid \Psi_{\mathbf{A}j}) = 0.5$ and $P(\Psi_{\mathbf{C}i} \mid \Psi_{\mathbf{B}j}) = 0.5$ for $i\ j$ in $\{\uparrow,\downarrow\}$.

The relevant equations admit no real-valued solutions. (Non-unique) solutions do exist if the "co-ordinate" domain is expanded to included complex numbers (a standard practice in mathematics over the centuries). The standard solution looks like:

$$\Psi_{\mathbf{C}\uparrow} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}, \Psi_{\mathbf{C}\downarrow} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

The geometric justification for this is that it is not possible to have 3 sets of right-angles in 2 dimensional Real space which all are at 45 degrees to each other; moving to complex 2 dimensional space provides the additional degree of freedom.

The corresponding standard measurement operators for Angular Momentum are

$$\mathbf{L}_z = 1/2.\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \mathbf{L}_x = 1/2.\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \mathbf{L}_y = 1/2.\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

If we add a 4th 2-valued conjugate quantity, then a solution requires quaternions. Re-solving to choose symmetric solutions, yields

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \pm i \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \pm j \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \pm k \end{bmatrix}$$

where ***i, j, k*** are quaternions. The corresponding operators are

$$\mathbf{K}_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \mathbf{K}_1 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}, \mathbf{K}_3 = \begin{bmatrix} 0 & -k \\ k & 0 \end{bmatrix}$$

Or replacing the quaternions $\boldsymbol{i} \rightarrow -i\sigma_x$, $\boldsymbol{j} \rightarrow -i\sigma_y$, $\boldsymbol{k} \rightarrow -i\sigma_z$ (and multiplying by -1) gives

$$\mathbf{K}_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \mathbf{K}_1 = \begin{bmatrix} 0 & -i.\sigma_x \\ i.\sigma_x & 0 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} 0 & -i.\sigma_y \\ i.\sigma_y & 0 \end{bmatrix}, \mathbf{K}_3 = \begin{bmatrix} 0 & -i.\sigma_z \\ i.\sigma_z & 0 \end{bmatrix}$$

which are recognizable as variants of the Dirac's matrices.

13. The First Ticker-Tape Conjecture

This conjecture is well-known but often only stated implicitly:

The commutator relationships between measurement operators define "what we are measuring".

The proposition has several advantages:

1. The algebraic relationships between operators can be extracted experimentally (with the usual caveats) from a measurement stream, even though this may be very expensive.

2. Commutators do not require Classical Physics to pre-define the concepts required for use by Quantum Mechanics.

3. Commutators naturally define a "scale" and provide a natural mechanism for the introduction of constants. For example, suppose that

   $$[\mathbf{X}_i, \mathbf{X}_j] = g_{ij}(\mathbf{X}_1 .. \mathbf{X}_n)$$

   If one of the devices, $\mathbf{X}_i$, is rescaled $\mathbf{X}_i \rightarrow f(\mathbf{X}_i)$ then, depending on the algebra, there may be a detectable change in the commutator relations

If there are n operators $\mathbf{X}_i$, then there are only $n(n\text{-}1)/2$ relationships from which to draw inferences. The RHS of the commutator cannot have infinite degrees of freedom and be able to be observational determined. The space is spanned by $\{\mathbf{X}_i | i=1..n\}$ so the simplest restriction would be to limit consideration to

$$[\mathbf{X}_i, \mathbf{X}_j] = const + c_{ij}^{\ k}\, \mathbf{X}_k, \quad i,j,k = 1..n.$$

14. Principle of Scale

A set of devices { $\mathbf{X}_i$, … $\mathbf{X}_n$ } *can be assigned meaning only if any rescaling* (*calibration*) *of the devices is detectable.*

The commuatator relations provide a method of ensuring different separated measuring devices use the same units.

<u>*Example 1*</u>: (Angular Momentum) Three devices are related by the commutator relationship

$$[\mathbf{L}_x, \mathbf{L}_y] = i\,\hbar\,\mathbf{L}_z$$
$$[\mathbf{L}_y, \mathbf{L}_z] = i\,\hbar\,\mathbf{L}_x$$
$$[\mathbf{L}_z, \mathbf{L}_x] = i\,\hbar\,\mathbf{L}_y$$

Rescale $\mathbf{L}_x \to \mathbf{L}_x' = \lambda\mathbf{L}_x$, $\lambda \neq 0$, $\lambda \neq 1$ then

$$[\mathbf{L}_x', \mathbf{L}_y] = i\,\lambda\,\hbar\,\mathbf{L}_z$$
$$[\mathbf{L}_y, \mathbf{L}_z] = i\,\hbar\left(\frac{1}{\lambda}\right)\mathbf{L}_x'$$
$$[\mathbf{L}_z, \mathbf{L}_x'] = i\,\lambda\,\hbar\,\mathbf{L}_y$$

The rescaling of $\mathbf{L}_x$ can be partially hidden by the simultaneous rescaling $\mathbf{L}_y$ or $\mathbf{L}_z$. If $\mathbf{L}_z \to \mathbf{L}_z' = \lambda\,\mathbf{L}_z$, then

$$[\mathbf{L}_x', \mathbf{L}_y] = i\,\hbar\,\mathbf{L}_z'$$
$$[\mathbf{L}_y, \mathbf{L}_z'] = i\,\hbar\,\mathbf{L}_z'$$
$$[\mathbf{L}_z', \mathbf{L}_x'] = i\,\lambda^2\,\hbar\,\mathbf{L}_y$$

However no further scaling $\mathbf{L}_y \to \mathbf{L}_y' = \mu\,\mathbf{L}_y$, $\mu \neq 0, \mu \neq 1$ (below) can disguise the original change of scale.

$$[\mathbf{L}_x', \mathbf{L}_y'] = i\,\mu\,\hbar\,\mathbf{L}_z'$$
$$[\mathbf{L}_y', \mathbf{L}_z'] = i\,\mu\,\hbar\,\mathbf{L}_z'$$
$$[\mathbf{L}_z', \mathbf{L}_x'] = i\,\lambda^2\left(\frac{1}{\mu}\right)\hbar\,\mathbf{L}_y'$$

Conclusion: A set of Mach devices described by angular momentum operators { $\mathbf{L}_x$, $\mathbf{L}_y$, $\mathbf{L}_z$ } satisfies the requirements of the Principle of Scale and can therefore be regarded as producing meaningful measurements.

Practically it means that 2 physically separated devices can determine that they are measuring the same physical quantitiy (angular momentum) and also tell that they are using the same units.

Example 2: (Position and Momentum) Two devices are related by commutator relationship $[\mathbf{x}, \mathbf{p}] = i\hbar$.

Rescale $\mathbf{x} \to \mathbf{x}' = \lambda\mathbf{x}$ (change in the choice of units), then

$$[\mathbf{x}', \mathbf{p}] = [\lambda\mathbf{x}, \mathbf{p}] = i\lambda\hbar.$$

The original scaling however can be hidden by rescaling $\mathbf{p}$. I.e. $\mathbf{p} \to \left(\frac{1}{\lambda}\right)\mathbf{p}$. In short, quantities are not naturally quantized.

Example 3: (Position and Momentum in 3D + Angular Momentum). There are nine devices in all. The non-zero commutator relations are shown below.

$[\mathbf{L}_x, \mathbf{L}_y] = i\,\hbar\,\mathbf{L}_z$	$[\mathbf{L}_y, \mathbf{L}_z] = i\,\hbar\,\mathbf{L}_x$	$[\mathbf{L}_z, \mathbf{L}_x] = i\,\hbar\,\mathbf{L}_y$
$[\mathbf{x}, \mathbf{p}_x] = i\,\hbar$	$[\mathbf{y}, \mathbf{p}_y] = i\,\hbar$	$[\mathbf{z}, \mathbf{p}_z] = i\,\hbar$
$[\mathbf{L}_x, \mathbf{y}] = i\,\hbar\,\mathbf{z}$	$[\mathbf{L}_y, \mathbf{z}] = i\,\hbar\,\mathbf{x}$	$[\mathbf{L}_z, \mathbf{x}] = i\,\hbar\,\mathbf{y}$
$[\mathbf{L}_x, \mathbf{p}_y] = i\,\hbar\,\mathbf{p}_z$	$[\mathbf{L}_y, \mathbf{p}_z] = i\,\hbar\,\mathbf{p}_x$	$[\mathbf{L}_z, \mathbf{p}_x] = i\,\hbar\,\mathbf{p}_y$

The set of Mach devices described by angular momentum + position + momentum operators does not satisfy the requirement of the Principle of Scale. That is not a surprise as it almost definitely indicates an incomplete description of space-time.

15. Time

Naive observers, equipped only with Mach devices, do not have access to a clock. How would such an observer measure time?

15.1 Pauli's Theorem

Pauli's Theorem strongly suggests that there is no such thing as a time operator in Quantum Mechanics. No-go theorems always need to be treated with caution. The best attempts at building a quantum clock so far are statistical in nature. We go with the assumption that that's as good as it gets.

15.2 Ensemble Clocks (Quantum Egg-Timers)

The diagram below shows an ensemble quantum clock frequently described in the literature *

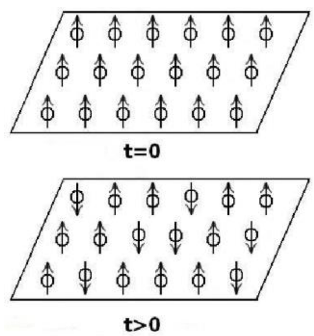

Figure 7
Quantum Clock.

The device consists of a large number of subsystems ("particles") that are initially prepared so that they are in identical states, denoted ↑. The "particles" spontaneously decay to a second state, denoted ↓. The transitions are not under the control of an observer, except perhaps that the observer may be able to switch the whole process off or on. Each transition is believed, from the analysis of transition probabilities, to be statistically independent from any other. The clock may also "reverse" so that the particles move back to the original state, and the whole cycle repeated (Flipping the Egg-Timer).

A statistical estimate of the time past since the assembly of particles was prepared (t = 0) can be made by counting the number of particles that have changed state (t > 0). A clock can be designed with arbitrarily high confidence by increasing the number of particles in the ensemble.

If $N(t)$ = expected proportion of "particles" in the ↑ state at time t, then

$$N(t) = e^{-\lambda t}$$

Solving for the time t:

$$t \approx - ln(N) / \lambda$$

Flipping the Egg-Timer also provides us with ability to accumulate statistics and check the operation of the clock. The individual particles that make up the clock should be independent of each other. All particles should be equally likely to make a transition "at any time". If there are N particles in the ensemble and the particles are ordered by order of transition, the average position for any particle should be N/2. A sieve can be constructed to exclude any particles that do not appear to be independent to within a specific confidence. Note however that this definition of time is local to a specific quantum system.

Once we have a clock, we can relax any requirement that transition probabilities are stable, and that always $P(\mathbf{\Psi} \mid \mathbf{\Psi}) = 1$. The Hamiltonian can be introduced by "differentiating" the state vector with respect to time.

* Type "ensemble quantum clock" into Google to see this.

16. The Second Ticker-Tape Conjecture

Time is a statistical concept.

The implication is that in extreme conditions, time may cease to be measurable and therefore the standard equations of Quantum Mechanics break down.

17. Conservation of Energy

Our definition of time is by definition homogenous. The connection with conservation of energy is well known. An isolated system (of quantum systems) is defined as a collection of quantum systems where the total energy is conserved.

Note that at this point there is no notion of space – and therefore there is no notion of an amplitude wave moving through space-time. Conservation of energy however allows us to abstract the concept of a particle "moving" between individual quantum systems, the same way as Chadwick inferred the existence of neutrons. The abstraction also allows the partition of quantum systems into measurement devices and a device-independent measured reality (e.g. If a measuring device is removed from the track of a particle, the particle will continue on its way unaffected)

18. Summary

The Ticker-Tape Interpretation is based on Copenhagen (I would hope that Bohr, Heisenberg and Born would recognise it and approve) but rejects the view of Bohr that classical notions of space-time and Physics are a necessary pre-requisite for the formulation of Quantum Physics.

It demonstrates that every-day properties of measurement lead to Quantum Mechanics as we know it. It is difficult to imagine how the Universe could be any other way – the mathematics of Quantum Mechanics are the outcome of analysing measurement streams along with taking practical steps to limit the scope of the analysis to that part of the Universe that can be simply understood. There is no need to invoke multiple universes, exotic strings or mysterious Bohmian trajectories.

19. References

[1] Copenhagen Interpretation of Quantum Mechanics, Stanford Encyclopaedia of Philosophy, Section 2 - Classical Physics, Jan 24 2004, http://plato.stanford.edu/entries/qm-copenhagen.

[2] Ariel Caticha, "Consistency and Linearity in Quantum Theory", arXiv:quant_ph/9803086 v1 31 Mar 1998.